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Target Setting in Production Technologies with Multiple Component Processes

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ABSTRACT

In this paper, we consider non-parametric production technologies with multiple component production processes, where each component uses both specific and shared inputs to produce specific and shared outputs. Investigating the issue of target setting within these technologies, we show that how efficient benchmarks for in-efficient units within the multi-component production technologies can be obtained by using basic envelopment and multiplier Data Envelopment Analysis models. Furthermore, we formulate efficient frontier of multi-component technology and develop a mixed integer optimization model that obtains closest targets for each in-efficient activity. The proposed models are finally illustrated on a real data set consisting 102 public universities in the UK.

Keywords: *Pareto-efficient, convexity, projection point, closest target.*

Introduction

Performance evaluation plays a central role in managing and improving production systems, particularly in competitive and resource-constrained industrial environments. Among the various approaches developed for efficiency analysis, Data Envelopment Analysis (DEA), originally introduced by Charnes, Cooper, and Rhodes (1978) based on the seminal work of Farrell (1957), has emerged as a widely adopted non-parametric method for assessing the relative efficiency of homogeneous decision-making units (DMUs) with multiple inputs and outputs. DEA constructs an empirical production possibility set using an axiomatic framework and represents the underlying technology as a closed convex polyhedral set. Based on this representation, an efficient frontier is identified against which all DMUs are evaluated. Inefficient units are projected onto this frontier to measure inefficiency and determine potential improvement targets.

Beyond efficiency measurement, target setting constitutes a fundamental component of DEA-based performance evaluation. This generally involves determining feasible and attainable performance levels for inefficient units through projecting them onto the efficient frontier. This issue has been studied by several researchers over the years. A comprehensive review of DEA applications in benchmarking can be found in Rostamzadeh et al. (2021). Traditionally, both radial and non-radial DEA models can be employed for this purpose. However, these models often generate projections that maximize the distance to the efficient frontier, which may result in unrealistic improvement plans requiring substantial and excessive input-output adjustments. To address this limitation, several path-based target setting approaches have been proposed, in which inefficient units follow a sequence of intermediate targets converging toward the efficient frontier (e.g. Lozano and Villa, 2005, 2010; Ghahraman and Prior 2016; Lozano and Calzada Infante, 2018, Dehnokhalaji and Soltani, 2019, Soltani and Lozano, 2020; Nasrabadi et al., 2018). More recently, increasing attention has been devoted to the concept of closest targets, where projection points are determined so as to minimize the distance between observed and target activities, thereby yielding more practical benchmarks for managerial action. Aparicio et al. (2007) formally characterized the efficient frontier of the standard DEA technology and developed a mixed-integer programming model to identify the closest target on the strongly efficient frontier. Their framework has been further extended in various directions, including group target setting (Cook et al., 2017), incorporation of expert preferences (Ruiz et al., 2015), common benchmarking and ranking (Ruiz and Sirvent, 2016), two-step benchmarking (Roman et al., 2018), monotone sequences of intermediate targets (Nasrabadi, 2019), balanced-effort benchmarking (Aparicio and Monge, 2022; Guevel et al., 2025), closest targets on extended facets of production possibility set (Zhu et al., 2022), and closer Pareto-efficient targets (Monge & Ruiz, 2023). Collectively, these studies have established closest-target setting as one of the most active research directions in DEA benchmarking, with increasing emphasis on generating realistic and practically attainable improvement paths for inefficient units.

Despite methodological advancements in DEA models and their wide applications in different sectors, most existing models have been developed within conventional production technologies, where each DMU is treated as a single aggregated production process. This assumption implicitly presumes that all inputs contribute to the production of all outputs, which may be overly restrictive in many real-world systems. However, this assumption seems to be too simplistic, as in many real-world applications the production process can be considered as a combination of multiple sub-processes, called components. For example, in numerous applications - such as universities, bank branches, or multi-service organizations -

production is more appropriately described as multiple interrelated components. Ignoring this internal structure may yield not only unreliable efficiency measures, but also impractical improvement plans that are technologically infeasible within the true underlying technology.

Early attempts to incorporate internal structures include Beasley (1995), who distinguished between teaching and research activities in universities, and Cook et al. (2000) as well as Cook and Zhu (2006, 2011), who studied bank branches with multiple functional components. Subsequently, a broad stream of research developed under the umbrella of Network DEA (see Kao, 2014, for a comprehensive review), typically modeling series or parallel structures. However, as it was mentioned by several researchers, the major challenge in modeling multicomponent technologies concerns the treatment of shared inputs and outputs, since it is generally unknown that what proportions of inputs and outputs have been consumed and produced by each component. For example, in the issue of efficiency analysis of schools or universities, the administration centralized expenditures should be treated as shared input, while the total number of students, or the number of published papers can be considered as shared outputs. Cook et al. (2000) and Cook and Green (2004) investigated this issue and proposed to consider the proportions of shared inputs as a variable that is optimistically determined in the process of efficiency analysis. Similar models were also developed in other researches, e.g. see Cook and Hababou (2001), Cook and Zhu (2011), Ding et al. (2015). In contrast, Cherchye et al. (2013, 2016) and Walheer (2018) tried to formulate multi-component technologies by assuming joint consumption or production of shared inputs and outputs. More recently, Podinovski et al. (2018) followed an axiomatic approach to formulate a multicomponent production technology based on a worst-case allocation principle. This framework was further refined in Podinovski (2022), who introduced a multicomponent convexity axiom allowing independent convexity coefficients for each component, thereby generalizing the standard convexity assumption. The scale characteristics of Podinovski (2022)'s model were later examined by Olesen et al. (2022). Moreover, a modified class of multicomponent technologies incorporating lower and upper bounds on the proportions of shared inputs and outputs to individual component processes was presented by Papaioannou and Podinovski (2023). Moreover, Free Disposal Hull models of multicomponent technologies were developed in Papaioannou and Podinovski (2025). As investigated in Podinovski (2022), the proposed axiomatic approach is capable for formulating both constant and variable returns to scale multi-component technologies, with the property that the obtained multicomponent CRS and VRS technologies contain the standard variations CRS and VRS as subsets. Hence, evaluation models formulated multicomponent technologies provide a better discrimination in terms of efficiency scores. These developments demonstrate the growing interest in multicomponent modeling and highlight the need for developing relevant aspects of these technologies.

While these contributions provide a rigorous foundation for modeling multicomponent production technologies, the issue of target setting within such frameworks remains largely unexplored. In practice, decision makers require improvement targets that are not only efficient under the conventional DEA technology but also feasible within the multicomponent structure governing the production process. Such targets derived from aggregated models may lie outside the feasible region of the corresponding multicomponent technology, may therefore be unreliable, since they can be infeasible within the underlying multicomponent technology, even though they appear superior to the evaluated unit in terms of the conventional efficiency score. This gap between structural modeling and target-setting methodology motivates the present study. In particular, to the best of our knowledge, no study has investigated the problem of identifying closest Pareto-efficient targets within the framework of multicomponent production technologies. This omission is important because targets obtained from conventional DEA technologies may not remain feasible or efficient once the internal structure of production processes is explicitly incorporated into the analysis.

This paper aims to fill the above research gap by developing a closest-target setting framework for multicomponent production technologies within DEA. The contributions of the study are threefold. First, we provide an explicit characterization of the Pareto-efficient frontier of the multicomponent production possibility set. Second, based on this characterization, we develop a mixed-integer optimization model capable of identifying the closest Pareto-efficient target for each inefficient DMU. Third, the proposed framework ensures that the resulting targets are not only efficient but also technologically feasible within the underlying multicomponent structure. Consequently, the proposed approach generates realistic and actionable benchmarks that require minimal performance adjustments. An empirical application is presented to illustrate the advantages of the proposed methodology over conventional DEA-based target-setting approaches.

The remainder of this paper is organized as follows. Section 2 reviews DEA as an axiomatic approach and introduces multicomponent technologies. Section 3 presents efficiency analysis models within the multicomponent framework and discusses their properties. Section 4 formulates the efficient frontier of multicomponent technologies and develops a closest-target model. Section 5 reports numerical and empirical results. Section 6 concludes the paper and present some directions for future research.

Multicomponent Production Technology

Consider a production technology in which m different inputs are consumed to produce s different outputs. The production technology is generally described as $T=\{(X,Y): Y \text{ can be produced from } X\}$. Each pair $(X,Y)\in T$ is called a feasible activity. Now, suppose we have n homogenous Decision-Making Units (DMUs), each consuming input vector of $X_j \in \mathbb{R}^m_+$ to produce output vector of $Y_j \in \mathbb{R}^s_+$. It is assumed that $X_j \neq \mathbf{0}$ and $Y_j \neq \mathbf{0}$, for all $j=1,...,n$. Each pair (X_j,Y_j) for $j=1,...,n$ is called an observed activity. Based on the input-output dataset and under the minimal extrapolation principle, the production possibility set in DEA is constructed using the following axioms:

P1 – (Feasibility of observations). All of the observed activities are feasible, i.e. for $j=1,...,n$, we have $(X_j,Y_j)\in T$.

P2 – (Free disposability). If $(X',Y')\in T$ and $(X'',-Y'')\geq(X',-Y')$, then $(X'',Y'')\in T$.

P3 – (Convexity). If $(X',Y'),(X'',Y'')\in T$, then $\mu(X',Y')+(1-\mu)(X'',Y'')\in T$ for all $\mu\in[0,1]$.

Accordingly, the DEA technology under Variable Returns to Scale (VRS) is formulated as:

$$T^{VRS}=\{(X,Y) \,|\, X\geq\sum_{j=1}^{n}\lambda_j X_j \text{ , } Y\leq\sum_{j=1}^{n}\lambda_j Y_j, \ \sum_{j=1}^{n}\lambda_j=1, \lambda_j\geq 0, \ j=1,...,n\}. \quad (1)$$

Now, consider a situation in which the production process in each DEA is composed of K components ($K\geq 2$). We decompose the index set of inputs $I=\{1,...,m\}$ and outputs $O=\{1,...,s\}$

respectively as $I = (\bigcup_{k=1}^{K} I_k) \cup I_S$, and $O = (\bigcup_{k=1}^{K} O_k) \cup O_S$, where I_k and O_k are the index sets for specific inputs and outputs of k-th component, for $k = 1,\dots,K$, and sets I_S and O_S denote the set of shared inputs and outputs, respectively. Assuming that each component has at least one input and at least one output, we have $I_k \cup I_s \neq \emptyset$ and $O_k \cup O_s \neq \emptyset$ for $k = 1,\dots,K$. Therefore, each DMU_j operating in this multi-component technology can be represented as $\begin{pmatrix} X_j \\ Y_j \end{pmatrix} = \begin{pmatrix} X_j^1, & \dots, & X_j^K, & X_j^S \\ Y_j^1, & \dots, & Y_j^K, & Y_j^S \end{pmatrix}$ for $j = 1,\dots,n$. It is assumed that each DMU_j has at least one positive input and one positive output for $j = 1,\dots,n$.

To define a multicomponent production technology, the convexity axiom **P3** should be modified in such a way that component-wise convex combinations are allowed. Following Podinovski (2022), we replace the standard convexity axiom **P3** with the following multicomponent convexity axiom:

P3* – (Multicomponent Convexity). For any set of non-negative K vectors $\boldsymbol{\lambda}^1,\dots,\boldsymbol{\lambda}^K \in \mathbb{R}_+^n$, such that $\mathbf{1}\boldsymbol{\lambda}^k = \sum_{j=1}^{n} \lambda_j^k = 1$ for $k = 1,\dots,K$, the multi-component activity $\begin{pmatrix} \hat{X} \\ \hat{Y} \end{pmatrix} = \begin{pmatrix} \hat{X}^1, & \dots, & \hat{X}^K, & \hat{X}^S \\ \hat{Y}^1, & \dots, & \hat{Y}^K, & \hat{Y}^S \end{pmatrix}$ defined as:

$$\begin{cases} \hat{X}^k = \sum_{j=1}^{n} \lambda_j^k X_j^k, \quad k = 1,\dots,K, \quad \hat{X}^S = \sum_{j=1}^{n} \max_{k=1,\dots,K}\{\lambda_j^k\} X_j^S, \\ \hat{Y}^k = \sum_{j=1}^{n} \lambda_j^k Y_j^k, \quad k = 1,\dots,K, \quad \hat{Y}^S = \sum_{j=1}^{n} \min_{k=1,\dots,K}\{\lambda_j^k\} Y_j^S, \end{cases} \tag{2}$$

belongs to T.

Now, considering the minimal extrapolation principle, the multi-component technology T^{MVRS} is defined as the intersection of all technologies $T \subseteq \mathbb{R}_{\geq 0}^{m+s}$ satisfying axioms **P1**, **P2**, and **P3*** (Podinovski 2022). This set is constructed as:

$$\begin{aligned} T^{MVRS} = \{ \begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} X^1,\dots,X^K,X^S \\ Y^1,\dots,Y^K,Y^S \end{pmatrix} \mid & X^k \geq \sum_{j=1}^{n} \lambda_j^k X_j^k, k = 1,\dots,K, X^S \geq \sum_{j=1}^{n} \max_{k=1,\dots,K}\{\lambda_j^k\} X_j^S, \\ & Y^k \leq \sum_{j=1}^{n} \lambda_j^k Y_j^k, k = 1,\dots,K, Y^S \leq \sum_{j=1}^{n} \min_{k=1,\dots,K}\{\lambda_j^k\} Y_j^S, \\ & \sum_{j=1}^{n} \lambda_j^k = 1, k = 1,\dots,K, \lambda_j^k \geq 0, j = 1,\dots,n, k = 1,\dots,K \}. \end{aligned} \tag{3}$$

It can be easily verified that T^{MVRS} can be equivalently presented as the following linearized form:

$$T^{MVRS} = \{ \begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} X^1, ..., X^K, X^S \\ Y^1, ..., Y^K, Y^S \end{pmatrix} \mid X^k \geq \sum_{j=1}^{n} \lambda_j^k X_j^k, k = 1, ..., K, X^S \geq \sum_{j=1}^{n} \lambda_j^{\max} X_j^S,$$

$$Y^k \leq \sum_{j=1}^{n} \lambda_j^k Y_j^k, k = 1, ..., K, Y^S \leq \sum_{j=1}^{n} \lambda_j^{\min} Y_j^S,$$

$$\lambda_j^{\max} \geq \lambda_j^k, \lambda_j^{\min} \leq \lambda_j^k, j = 1, ..., n, k = 1, ..., K,$$

$$\sum_{j=1}^{n} \lambda_j^k = 1, k = 1, ..., K,$$

$$\lambda_j^{\max}, \lambda_j^{\min}, \lambda_j^k \geq 0, j = 1, ..., n, k = 1, ..., K \}. \qquad (4)$$

Moreover, as shown in Podinovski (2022), in the special case that $\boldsymbol{\lambda}^1 = ... = \boldsymbol{\lambda}^K = \boldsymbol{\lambda}$ the set T^{MVRS} reduces to the standard DEA technology T^{VRS}, given in (1). This proves that $T^{VRS} \subseteq T^{MVRS}$. The concept of Pareto-efficiency in T^{MVRS} is defined as follows.

Definition 1. A multi-component activity $\begin{pmatrix} \bar{X} \\ \bar{Y} \end{pmatrix} = \begin{pmatrix} \bar{X}^1, ..., \bar{X}^K, \bar{X}^S \\ \bar{Y}^1, ..., \bar{Y}^K, \bar{Y}^S \end{pmatrix} \in T^{MVRS}$ is called Parto-efficient in T^{MVRS} iff there does not exist any $\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} X^1, ..., X^K, X^S \\ Y^1, ..., Y^K, Y^S \end{pmatrix} \in T^{MVRS}$ such that $\begin{pmatrix} -X \\ Y \end{pmatrix} \geq \begin{pmatrix} -\bar{X} \\ \bar{Y} \end{pmatrix}$, and $\begin{pmatrix} -X \\ Y \end{pmatrix} \neq \begin{pmatrix} -\bar{X} \\ \bar{Y} \end{pmatrix}$.

To evaluate the performance of an observed activity DMU_o ($o \in \{1, ..., n\}$) w.r.t. the technology set T^{MVRS}, and provide benchmark points for each inefficient unit, relevant DEA-based models can be formulated and solved. Podinovski (2022) proposed a multi-component radial model in output orientation. In next section, we formulate a basic additive model for multicomponent processes and then we discuss the concept of benchmarking for these DEA models, i.e. the radial model, and the basic additive model.

DEA-based Models in Multicomponent Technologies

In this section, we investigate the issue of target setting for inefficient activities in multicomponent technologies. Our aim is to provide Pareto-efficient projections and benchmarking targets that are both feasible and actionable. We first consider the radial model, which focuses on proportional improvement of outputs, and then introduce an additive model, which explicitly captures input-output trade-offs across multiple components. Therefore, we obtain a robust framework for performance improvement and decision support.

Radial Model

Based on the formulation of T^{MVRS} as given in (4), the output oriented radial model for evaluating DMU_o is formulated as (Podinovski, 2022):

$$
\begin{aligned}
\varphi^* = \max \quad & \varphi \qquad\qquad (5)\\
\text{s.t.} \quad & X_o^k \geq \sum_{j=1}^{n} \lambda_j^k X_j^k, \qquad k=1,\dots,K,\\
& X_o^S \geq \sum_{j=1}^{n} \lambda_j^{\max} X_j^S,\\
& \varphi Y_o^k \leq \sum_{j=1}^{n} \lambda_j^k Y_j^k, \qquad k=1,\dots,K,\\
& \varphi Y_o^S \leq \sum_{j=1}^{n} \lambda_j^{\min} Y_j^S,\\
& \lambda_j^{\max} \geq \lambda_j^k, \lambda_j^{\min} \leq \lambda_j^k, \qquad j=1,\dots,n, k=1,\dots,K,\\
& \sum_{j=1}^{n} \lambda_j^k = 1, \qquad k=1,\dots,K,\\
& \lambda_j^{\max}, \lambda_j^{\min}, \lambda_j^k \geq 0, \qquad j=1,\dots,n, k=1,\dots,K.
\end{aligned}
$$

It can be easily verified that $\varphi_o^* \geq 1$. If $\varphi_o^* = 1$, then DMU_o is weakly efficient, and lies on the frontier of T^{MVRS}. To verify whether DMU_o is strong (or Pareto) efficient, it is required to check the possibility of any positive slack. Therefore, the Phase II model is formulated by imposing $\varphi = \varphi_o^*$ in the constraint of the above model, restating all constraints as equalities by introducing slack variables, and maximizing the sum of all slacks (Cooper et al, 2007). Any optimal solution for the Phase II model corresponding to model (5) is called a max-slack solution for model (5). Clearly, DMU_o is Pareto-efficient in T^{MVRS} iff $\varphi_o^* = 1$ and all slacks are zero in any max-slack solution for model (5). Otherwise, DMU_o is called inefficient.

The dual form of model (5), called the multiplier output radial model is formulated as:

$$
\begin{aligned}
\varphi_o^* = \min \quad & \sum_{k=1}^{K} V^k X_o^k - V^S X_o^S + \sum_{k=1}^{K} w^k \qquad\qquad (6)\\
\text{s.t.} \quad & \sum_{k=1}^{K} U^k Y_o^k + U^S Y_o^S = 1\\
& V^k X_j^k - U^k Y_j^k + w^k + \pi_j^k - \gamma_j^k \geq 0, \qquad j=1,\dots,n, k=1,\dots,K,\\
& -U^S Y_j^S + \sum_{k=1}^{K} \gamma_j^k = 0, \qquad j=1,\dots,n,\\
& V^S X_j^S - \sum_{k=1}^{K} \pi_j^k = 0, \qquad j=1,\dots,n,\\
& U^k \geq \mathbf{0}, V^k \geq \mathbf{0}, w^k \ \text{free} \qquad k=1,\dots,K,\\
& U^S \geq \mathbf{0}, V^S \geq \mathbf{0},\\
& \pi_j^k, \gamma_j^k \geq 0, \qquad j=1,\dots,n, k=1,\dots,K.
\end{aligned}
$$

It can be easily shown that DMU_o is Pareto-efficient iff the optimal value of the multiplier model (6) is equal to 1, and all weights U 's and V 's are strictly positive in some optimal solution.

The following definition shows that how a projection point can be defined based on a max-slack solution of the envelopment model (5).

Definition 2. Assuming that $(\varphi_o^*, \boldsymbol{\lambda}^*)$ is part of a max-slack solution for model (5), the projection point for DMU_o is defined as:

$$\begin{cases} \hat{X}_o^k = \sum_{j=1}^{n} \lambda_j^{k*} X_j^k, \quad k = 1, \dots, K, \quad \hat{X}_o^S = \sum_{j=1}^{n} \lambda_j^{\max *} X_j^S, \\ \hat{Y}_o^k = \sum_{j=1}^{n} \lambda_j^{k*} Y_j^k, \quad k = 1, \dots, K, \quad \hat{Y}_o^S = \sum_{j=1}^{n} \lambda_j^{\min *} Y_j^S, \end{cases} \tag{7}$$

Obviously, if DMU_o is Pareto efficient, then the projection point (7) coincides DMU_o, otherwise it dominates DMU_o. Generally, the obtained projection point is Pareto-efficient, as established in the following.

Proposition 1. The projection point for DMU_o as defined in (7) is Pareto-efficient.

Proof. To prove that the projection point (7) is Pareto-efficient, we evaluate this activity by implementing the two-phase approach, as described above. Recalling that the projection point (7) is defined based on a max-slack solution, i.e. after solving Phase I and Phase II models for DMU_o, it can be easily verified that, the optimal value of the Phase I model evaluating the projection point, is equal to one, and that of the Phase II model is zero. This means that the projection point (7) is Pareto-efficient, and we are done.

The following proposition is a straightforward result of the principle of minimax extrapolation. However, it can be proved in an alternative way.

Proposition 2. There exists $p \in \{1, \dots, n\}$ such that DMU_p is Pareto-efficient.

Proof. Suppose by contrary that for each $j, (j = 1, \dots, n)$ DMU_j is Pareto-inefficient. Then, based on the definition of Pareto efficiency as given in Definition 1, it is concluded that the projection point (7), which is in fact a multicomponent convex combination of DMU_j 's for $j = 1, \dots, n$, is also Pareto-inefficient. This contradicts Proposition 1, and the proof is complete.

The following proposition proves that any activity having a strictly positive coefficient in the projection point defined in (7), is also Pareto-efficient.

Proposition 3. Let DMU_p be such that $\lambda_p^{\min *} > 0$, where $(\varphi_o^*, \boldsymbol{\lambda}^*)$ is a max-slack solution for model (5). Then, DMU_p is Pareto-efficient in T^{MVRS}.

Proof. Let $(\varphi_o^*, \boldsymbol{\lambda}^*)$ be a max-slack solution for model (5). Then, $\lambda_p^{\min *} > 0$ for some $p \in \{1, \dots, n\}$. Since $\lambda_p^{\min *} > 0$, we have $\lambda_p^{k*} > 0$ for $k = 1, \dots, K$. This implies that there exists an optimal solution for model (6) evaluating DMU_o at which all constraints $V^k X_j^k - U^k Y_j^k + w^k + \pi_j^k - \gamma_j^k \geq 0$, for $k = 1, \dots, K$ are active. Assuming that $\sum_{k=1}^{K} U^{k*} Y_p^k + U^{S*} Y_p^S = \alpha > 0$, it can be verified that $\frac{1}{\alpha}\left[(U^{k*}, V^{k*}, w^{k*}, \boldsymbol{\pi}^{k*}, \boldsymbol{\gamma}^{k*})_{k=1,\dots,K}, (U^{S*}, V^{S*}) \right]$ is a feasible solution for multiplier model (6) evaluating DMU_p, with objective value of one, and hence it is optimal. This completes the proof.

Based on the above proposition, the reference set for DMU_o can be defined as $R_o = \{ p \mid \lambda_p^{\min *} > 0 \}$ where $\boldsymbol{\lambda}^*$ corresponds to a max-slack solution for model (5).

Additive Model

While the radial model focuses on proportional output expansion, the additive model captures individual input-output adjustments and explicitly measures slack across components. Let DMU_o be the unit under evaluation, where $o \in \{1,\dots,n\}$. Considering the formulation of T^{MVRS} as given in (4), we formulate the additive model for evaluating DMU_o as:

$$
\begin{aligned}
\delta_o^* = \max \quad & \sum_{k=1}^{K} \mathbf{1}T^{-k} + \mathbf{1}T^{-S} + \sum_{k=1}^{K} \mathbf{1}T^{+k} + \mathbf{1}T^{+S} && (13)\\
\text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j^k X_j^k + T^{-k} = X_o^k, && k = 1,\dots,K,\\
& \sum_{j=1}^{n} \lambda_j^{\max} X_j^S + T^{-S} = X_o^S, &&\\
& \sum_{j=1}^{n} \lambda_j^k Y_j^k - T^{+k} = Y_o^k, && k = 1,\dots,K,\\
& \sum_{j=1}^{n} \lambda_j^{\min} Y_j^S - T^{+S} = Y_o^S, &&\\
& \lambda_j^{\max} \ge \lambda_j^k, \lambda_j^{\min} \le \lambda_j^k, && j = 1,\dots,n, k = 1,\dots,K,\\
& \sum_{j=1}^{n} \lambda_j^k = 1, && k = 1,\dots,K,\\
& \lambda_j^{\max}, \lambda_j^{\min}, \lambda_j^k \ge 0, && j = 1,\dots,n, k = 1,\dots,K,\\
& (T^{-S}, T^{+S}) \ge (\mathbf{0},\mathbf{0}), (T^{-k}, T^{+k}) \ge (\mathbf{0},\mathbf{0}), && k = 1,\dots,K.
\end{aligned}
$$

The following theorem can be established in a straightforward way.

Theorem 1. DMU_o is Pareto-efficient in T^{MVRS} iff $\delta_o^* = 0$, or equivalently all slacks are zero at any optimal solution of (8).

Proof. Straightforward, based on the definition of Pareto-efficiency.

The multi-component additive model (13) can be suitably used for the aim of benchmarking.

Definition 2. For a Pareto-inefficient DMU_o, its additive-based projection point is defined as:

$$\begin{cases} \hat{X}_o^k = \sum_{j=1}^{n} \lambda_j^{k*} X_j^k = X_o^k - T^{-k*} \quad k=1,\dots,K, \quad \hat{X}_o^S = \sum_{j=1}^{n} \lambda_j^{\max*} X_j^S = X_o^S - T^{-S*}, \\ \hat{Y}_o^k = \sum_{j=1}^{n} \lambda_j^{k*} Y_j^k = Y_o^k - T^{+k*}, \quad k=1,\dots,K, \quad \hat{Y}_o^S = \sum_{j=1}^{n} \lambda_j^{\min*} Y_j^S = Y_o^S - T^{+S*}, \end{cases} \tag{14}$$

where $(\lambda^*, (T^{-k*}, T^{-k*})_{k=1,\dots,K}, T^{-S*}, T^{+S*})$ is an optimal solution for the additive model (13). Clearly, if DMU_o is Pareto inefficient, then this projection point dominates DMU_o.

Theorem 2. The additive-based projection point defined in (14), is Pareto-efficient in T^{MVRS}.

Proof. To show that the projection point (14) is Pareto-efficient in T^{MVRS}, we evaluate this activity by the basic additive model (13). Doing so, it can be easily verified that all slack variables should be zero at optimality, and this completes the proof.

The additive model formulated in (13) has an envelopment form. Its dual form, called the multiplier additive model is formulated as:

$$\begin{aligned} \delta_o^* = \max \quad & \sum_{k=1}^{K} U^k Y_o^k + U^S Y_o^S - \sum_{k=1}^{K} V^k X_o^k - V^S X_o^S + \sum_{k=1}^{K} w^k && (15) \\ \text{s.t.} \quad & U^k Y_j^k - V^k X_j^k + w^k - \pi_j^k + \gamma_j^k \le 0, && j=1,\dots,n, k=1,\dots,K, \\ & U^S Y_j^S - \sum_{k=1}^{K} \gamma_j^k = 0, && j=1,\dots,n, \\ & V^S X_j^S - \sum_{k=1}^{K} \pi_j^k = 0, && j=1,\dots,n, \\ & U^k \ge \mathbf{1}, V^k \ge \mathbf{1}, w^k \quad \text{free}, && k=1,\dots,K, \\ & U^S \ge \mathbf{1}, V^S \ge \mathbf{1}, \\ & \pi_j^k, \gamma_j^k \ge 0, && j=1,\dots,n, k=1,\dots,K. \end{aligned}$$

The following proposition can be established similar to Proposition 3.

Proposition 4. Let DMU_p be such that $\lambda_p^{\min*} > 0$, where λ^* is an optimal solution for the additive model (13). Then, DMU_p is Pareto-efficient in T^{MVRS}.

Similarly, the reference set for DMU_o can be defined based on the additive model as $R_o = \{p \mid \lambda_p^{\min*} > 0\}$, where $\boldsymbol{\lambda}^*$ is an optimal solution for model (13).

The primal-dual relationships between models (13) and (15), along with Complementarity Slackness (CS) conditions establish a powerful framework for finding closest targets which ensures projections to be as close as possible to an observed inefficient unit, while respecting the multicomponent technology. To explain, we first obtain a comprehensive formulation of the Pareto-efficient frontier of the multicomponent technology, in the next section which serves as a foundation for developing a closest target model.

Efficient Frontier and Closest Targets

In this section, we develop a unified framework for identifying Pareto-efficient benchmarks and finding the closest feasible targets for inefficient units in multicomponent production technologies. Based on the multicomponent additive model formulated in both envelopment and multiplier forms (13) and (15), drawing on the methodology proposed by Aparicio et al. (2007), we first formulate the efficient frontier of T^{MVRS}. We then make use of this frontier to construct a closest target model, ensuring that obtained benchmarks are both feasible and practically implantable.

Efficient Frontier of Multicomponent Technology

The efficient frontier represents all Pareto-efficient activities within the multicomponent production possibility set. Constructing this frontier is essential for target setting, as it allows us to identify Pareto-efficient performance benchmarks for each inefficient unit while respecting the internal structure of the production process. While radial and additive projections presented in Section 3 provide feasible targets for inefficient units, these projections do not necessarily minimize the distance between the observed performance and the efficient frontier. In practical terms, decision-makers often prefer targets that are not only technologically feasible and Pareto-efficient, but also as close as possible to current operations, to reduce the implementation effort and ensure realistic adjustments in inputs and outputs.

In this regard, finding an explicit formulation of the efficient frontier is essential for developing a closest-target model, which ensures that each inefficient DMU is projected onto a Pareto-efficient activity that is nearest in the multicomponent input-output space. This approach enhances the practical applicability of the DEA framework by bridging the gap between theoretical efficiency analysis and credible managerial decisions. The following theorem provides a formal mathematical representation of this efficient frontier.

Theorem 3. The efficient frontier of T^{MVRS} is formulated as:

$$
\begin{aligned}
T_E^{MVRS} = \{\begin{pmatrix} X^1,\ldots,X^K,X^S \\ Y^1,\ldots,Y^K,Y^S \end{pmatrix} \mid\ & X^k = \sum_{j\in E}\lambda_j^k X_j^k, k=1,\ldots,K, X^S = \sum_{j\in E}\lambda_j^{\max} X_j^S, \\
& Y^k = \sum_{j\in E}\lambda_j^k Y_j^k, k=1,\ldots,K, Y^S = \sum_{j\in E}\lambda_j^{\max} Y_j^S, \\
& \sum_{j\in E}\lambda_j^k = 1, && k=1,\ldots,K, \\
& \lambda_j^{\max} \ge \lambda_j^k, \lambda_j^{\min} \le \lambda_j^k, && j\in E, k=1,\ldots,K, \\
& U^k Y_j^k - V^k X_j^k + w^k - \pi_j^k + \gamma_j^k + d_j^k = 0, && j\in E, k=1,\ldots,K, \\
& U^S Y_j^S - \sum_{k=1}^{K}\gamma_j^k = 0,\ V^S X_j^S - \sum_{k=1}^{K}\pi_j^k = 0, && j\in E, \\
& \lambda_j^k d_j^k = 0, && j\in E, k=1,\ldots,K, \\
& \pi_j^k(\lambda_j^{\max} - \lambda_j^k) = 0,\ \gamma_j^k(\lambda_j^k - \lambda_j^{\min}) = 0, && j\in E, k=1,\ldots,K, \\
& U^S \ge \mathbf{1}, V^S \ge \mathbf{1}, U^k \ge \mathbf{1}, V^k \ge \mathbf{1}, w^k \ \text{ free} && k=1,\ldots,K, \\
& \pi_j^k, \gamma_j^k, \lambda_j^k, \lambda_j^{\max}, \lambda_j^{\min} \ge 0, && j\in E, k=1,\ldots,K\}, \qquad (16)
\end{aligned}
$$

where $E \subseteq \{1,\ldots,n\}$ is the index set of all Pareto-efficient DMUs in T^{MVRS}, i.e. $E = \{j \mid \delta_j^* = 0\}$.

Proof. Let $\begin{pmatrix} \tilde{X}^1,\ldots,\tilde{X}^K,\tilde{X}^S \\ \tilde{Y}^1,\ldots,\tilde{Y}^K,\tilde{Y}^S \end{pmatrix} \in T^{MVRS}$ be a Pareto-efficient activity in T^{MVRS}. Hence, the optimal value of the following pair of primal-dual LP problems is equal to zero:

$$\delta_o^* = \max \quad \sum_{k=1}^{K} \mathbf{1}T^{-k} + \mathbf{1}T^{-S} + \sum_{k=1}^{K} \mathbf{1}T^{+k} + \mathbf{1}T^{+S} \tag{17}$$

$$\begin{aligned}
\text{s.t.} \quad & \sum_{j\in E} \lambda_j^k X_j^k + T^{-k} = \tilde{X}^k, && k = 1,\ldots,K,\\
& \sum_{j\in E} \lambda_j^{\max} X_j^S + T^{-S} = \tilde{X}^S, &&\\
& \sum_{j\in E} \lambda_j^k Y_j^k - T^{+k} = \tilde{Y}^k, && k = 1,\ldots,K,\\
& \sum_{j\in E} \lambda_j^{\min} Y_j^S - T^{+S} = \tilde{Y}^S, &&\\
& \lambda_j^{\max} \ge \lambda_j^k, \lambda_j^{\min} \le \lambda_j^k, && j \in E, k = 1,\ldots,K,\\
& \sum_{j\in E} \lambda_j^k = 1, && k = 1,\ldots,K,\\
& \lambda_j^{\max}, \lambda_j^{\min}, \lambda_j^k \ge 0, && j \in E, k = 1,\ldots,K,\\
& (T^{-S}, T^{+S}) \ge (\mathbf{0},\mathbf{0}), (T^{-k}, T^{+k}) \ge (\mathbf{0},\mathbf{0}), && k = 1,\ldots,K.
\end{aligned}$$

and

$$\delta_o^* = \max \quad \sum_{k=1}^{K} U^k \tilde{Y}^k + U^S \tilde{Y}^S - \sum_{k=1}^{K} V^k \tilde{X}^k - V^S \tilde{X}^S + \sum_{k=1}^{K} w^k \tag{18}$$

$$\begin{aligned}
\text{s.t.} \quad & U^k Y_j^k - V^k X_j^k + w^k - \pi_j^k + \gamma_j^k \le 0, && j \in E, k = 1,\ldots,K,\\
& U^S Y_j^S - \sum_{k=1}^{K} \gamma_j^k \le 0, && j = 1,\ldots,n,\\
& V^S X_j^S - \sum_{k=1}^{K} \pi_j^k \le 0, && j = 1,\ldots,n,\\
& U^k \ge \mathbf{1}, V^k \ge \mathbf{1}, w^k \quad \text{free}, && k = 1,\ldots,K,\\
& U^S \ge \mathbf{1}, V^S \ge \mathbf{1}, &&\\
& \pi_j^k, \gamma_j^k \ge 0, && j \in E, k = 1,\ldots,K.
\end{aligned}$$

Since $\begin{pmatrix} \tilde{X}^1,\ldots,\tilde{X}^K,\tilde{X}^S \\ \tilde{Y}^1,\ldots,\tilde{Y}^K,\tilde{Y}^S \end{pmatrix}$ is Pareto-efficient in T^{MVRS}, there exist $(\tilde{U}^S, \tilde{V}^S, (\tilde{U}^k, \tilde{V}^k, \tilde{w}^k)_{k=1,\ldots,K}, (\tilde{\pi}_j^k)_{j\in E,k=1,\ldots,K}, (\tilde{\gamma}_j^k)_{j\in E,k=1,\ldots,K})$, and $(\tilde{\lambda}_j^k)_{j\in E,k=1,\ldots,K} \ge 0$, such that

$$\begin{cases}
\sum_{j\in E} \tilde{\lambda}_j^k X_j^k = \tilde{X}^k, & k = 1,\ldots,K,\\
\sum_{j\in E} \tilde{\lambda}_j^{\max} X_j^S = \tilde{X}^S, &\\
\sum_{j\in E} \tilde{\lambda}_j^k Y_j^k = \tilde{Y}^k, & k = 1,\ldots,K,\\
\sum_{j\in E} \tilde{\lambda}_j^{\min} Y_j^S = \tilde{Y}^S, &\\
\tilde{\lambda}_j^{\max} \ge \tilde{\lambda}_j^k, \tilde{\lambda}_j^{\min} \le \tilde{\lambda}_j^k, & j \in E, k = 1,\ldots,K,\\
\sum_{j\in E} \tilde{\lambda}_j^k = 1, & k = 1,\ldots,K,
\end{cases} \tag{19}$$

and

$$\begin{cases} \tilde{U}^k Y_j^k - \tilde{V}^k X_j^k + \tilde{w}^k - \tilde{\pi}_j^k + \tilde{\gamma}_j^k \le 0, & j \in E, k = 1, \dots, K, \\ \tilde{U}^S Y_j^S - \sum_{k=1}^{K} \tilde{\gamma}_j^k = 0, & j \in E, \\ V^S X_j^S - \sum_{k=1}^{K} \tilde{\pi}_j^k = 0, & j \in E, \\ \tilde{U}^k \ge \mathbf{1}, \tilde{V}^k \ge \mathbf{1}, \tilde{w}^k \quad \text{free}, & k = 1, \dots, K, \\ \tilde{U}^S \ge \mathbf{1}, \tilde{V}^S \ge \mathbf{1}, & \\ \tilde{\pi}_j^k, \tilde{\gamma}_j^k \ge 0, & j \in E, k = 1, \dots, K. \end{cases} \tag{20}$$

Moreover, by complementarity slackness conditions, for each $j \in E, k = 1, \dots, K$, we have:

$$\begin{cases} \tilde{\lambda}_j^k (\tilde{U}^k Y_j^k - \tilde{V}^k X_j^k + \tilde{w}^k - \tilde{\pi}_j^k + \tilde{\gamma}_j^k) = 0, \\ \tilde{\pi}_j^k (\tilde{\lambda}_j^{\max} - \tilde{\lambda}_j^k) = 0, \\ \tilde{\gamma}_j^k (\tilde{\lambda}_j^{\min} - \tilde{\lambda}_j^k) = 0. \end{cases} \tag{21}$$

Therefore, it is concluded that $\begin{pmatrix} \tilde{X}^1, \dots, \tilde{X}^K, \tilde{X}^S \\ \tilde{Y}^1, \dots, \tilde{Y}^K, \tilde{Y}^S \end{pmatrix} \in T_E^{MVRS}$. On the other hand, if $\begin{pmatrix} \tilde{X}^1, \dots, \tilde{X}^K, \tilde{X}^S \\ \tilde{Y}^1, \dots, \tilde{Y}^K, \tilde{Y}^S \end{pmatrix} \in T_E^{MVRS}$ is arbitrary, then by a similar argument it can be verified that this activity is Pareto-efficient in T^{MVRS}, and the proof is complete.

The explicit formulation of the efficient frontier of T^{MVRS}, as given in (16), allows us to evaluate any inefficient unit against the full set of Pareto-efficient activities, and find the closest point on this set as the corresponding benchmark. These benchmarks reflect both the internal structure of the production system and practical feasibility. In fact, as we see in the next subsection, the frontier points form a feasible set for the closest-target model, bridging theoretical efficiency analysis with implementable decision-making.

Finding Closest Targets for Inefficient Units

Let DMU_o be Pareto inefficient (i.e. $o \notin E$), To determine closest feasible target for DMU_o, we formulate an optimization problem that minimizes the distance between the observed activity and the frontier while preserving Pareto-efficiency in terms of L_1-distance, Formally, the model is formulated as:

$$\min \quad \sum_{k=1}^{K} \mathbf{1}T^{-k} + \mathbf{1}T^{-S} + \sum_{k=1}^{K} \mathbf{1}T^{+k} + \mathbf{1}T^{+S} \tag{22}$$

$$\begin{array}{llll}
\text{s.t.} & \sum_{j\in E} \lambda_j^k X_j^k + T^{-k} = X_o^k, & k=1,\dots,K, & \text{(a)} \\
& \sum_{j\in E} \lambda_j^{\max} X_j^S + T^{-S} = X_o^S, & & \text{(b)} \\
& \sum_{j\in E} \lambda_j^k Y_j^k - T^{+k} = Y_o^k, & k=1,\dots,K, & \text{(c)} \\
& \sum_{j\in E} \lambda_j^{\min} Y_j^S - T^{+S} = Y_o^S, & & \text{(d)} \\
& \sum_{j\in E} \lambda_j^k = 1, & k=1,\dots,K, & \text{(e)} \\
& \lambda_j^{\max} \ge \lambda_j^k, \lambda_j^{\min} \le \lambda_j^k, & j\in E, k=1,\dots,K, & \text{(f)} \\
& U^k Y_j^k - V^k X_j^k + w^k - \pi_j^k + \gamma_j^k + d_j^k = 0, & j\in E, k=1,\dots,K, & \text{(g)} \\
& U^S Y_j^S - \sum_{k=1}^{K} \gamma_j^k = 0,\ V^S X_j^S - \sum_{k=1}^{K} \pi_j^k = 0, & j\in E, & \text{(h)} \\
& \lambda_j^k d_j^k = 0, & j\in E, k=1,\dots,K, & \text{(i)} \\
& \pi_j^k (\lambda_j^{\max} - \lambda_j^k) = 0, & j\in E, k=1,\dots,K, & \text{(j)} \\
& \gamma_j^k (\lambda_j^k - \lambda_j^{\min}) = 0, & j\in E, k=1,\dots,K, & \text{(k)} \\
& U^S \ge \mathbf{1}, V^S \ge \mathbf{1}, U^k \ge \mathbf{1}, V^k \ge \mathbf{1}, w^k \quad \text{free}, & k=1,\dots,K, & \text{(l)} \\
& \pi_j^k, \gamma_j^k, \lambda_j^k, \lambda_j^{\max}, \lambda_j^{\min} \ge 0, & j\in E, k=1,\dots,K, & \text{(m)} \\
& (T^{-S}, T^{+S}) \ge (\mathbf{0},\mathbf{0}), (T^{-k}, T^{+k}) \ge (\mathbf{0},\mathbf{0}), & k=1,\dots,K. & \text{(n)}
\end{array}$$

Assuming that $(\lambda^*, (T^{-k*}, T^{-k*})_{k=1,\dots,K}, T^{-S*}, T^{+S*})$ is part of an optimal solution for the above model (22), the corresponding closest target for DMU_o is obtained as:

$$\begin{cases} \hat{X}_o^k = \sum_{j\in E} \lambda_j^{k*} X_j^k = X_o^k - T^{-k*} \quad k=1,\dots,K, \quad \hat{X}_o^S = \sum_{j\in E} \lambda_j^{\max *} X_j^S = X_o^S - T^{-S*}, \\ \hat{Y}_o^k = \sum_{j\in E} \lambda_j^{k*} Y_j^k = \mathrm{Y}_o^k - T^{+k*}, \quad k=1,\dots,K, \quad \hat{Y}_o^S = \sum_{j\in E} \lambda_j^{\min *} Y_j^S = Y_o^S - T^{+S*}, \end{cases} \tag{23}$$

It can be observed that this activity lies on the multicomponent efficient frontier, ensuring adherence to production constraints. Moreover, the projected target dominates the original inefficient activity, maintaining strong efficiency, while minimizing the deviation from observed inputs and outputs ensures a realistic and implementable benchmarks.

Remark. The non-linear constrains (i), (j), and (k) in model (22) can be converted to the following equivalent zero-one constraints, respectively as follow:

$$\begin{cases} \lambda_j^k \le M l_j^k, d_j^k \le M(1-l_j^k), & l_j^k \in \{0,1\} \quad j\in E, k=1,\dots,K, \quad \text{(i')} \\ \pi_j^k \le M f_j^k, \lambda_j^{\max} - \lambda_j^k \le M(1-f_j^k), & f_j^k \in \{0,1\} \quad j\in E, k=1,\dots,K, \quad \text{(j')} \\ \gamma_j^k \le M h_j^k, \lambda_j^k - \lambda_j^{\min} \le M(1-h_j^k), & h_j^k \in \{0,1\} \quad j\in E, k=1,\dots,K, \quad \text{(k')} \end{cases} \tag{24}$$

where $M > 0$ is a sufficiently large positive number. Formulating these three constraints as above, leads to a mixed integer linear program with $6K \times |E|$ zero-one variables. However, in practice, it is possible to solve model (22) by using SOS-type 1 variables. To explain, for each pair $(j,k)_{j\in E,k=1,\dots,K}$, each of the three sets $\{\lambda_j^k, d_j^k\}$, $\{\pi_j^k, \lambda_j^{\max}-\lambda_j^k\}$, and $\{\gamma_j^k, \lambda_j^k-\lambda_j^{\min}\}$ is considered as SOS1-variable, meaning that at most one of the two variables in each set is allowed to take positive value. Using SOS-1 variables for modelling the above complementarity non-linear variables is more practical and less challenging than using the big-M approach.

However, the computational complexity of model (22) requires careful consideration. The model involves a large number of constraints as well as three sets of SOS1 variables, reflecting the discrete selection of reference activities and the component-wise allocation of inputs and outputs. As a result, solving the model can be computationally demanding, especially for datasets with many decision-making units or multiple components. Despite this challenge, the formulation provides substantial practical value. By explicitly ensuring that the closest target lies on the Pareto-efficient frontier, the model generates effective and realistic benchmarks that are both feasible and minimal in terms of adjustment from current operations. To mitigate computational difficulties, modern optimization solvers can exploit the SOS1 structure and problem-specific properties, allowing the approach to be applied effectively in medium- to large-scale applications. An empirical application of the proposed model is illustrated in next section.

Application

In this section, we illustrate the proposed target setting approach using a real-world application. The data set, drawn from Podinovski (2022), consists a set of 102 public universities in the United Kingdom (UK) each offering both undergraduate and postgraduate programs across three areas: Medicine, Science, and Social Sciences. This structure naturally allows us to consider each university as a combination of three component processes. For each component, the total expenditure (including academic and non-academic staff costs as well as other expenditures) is considered as the single component-specific input. The number of undergraduate and postgraduate students serves as the component-specific outputs. In addition, centralized expenditures and number of published papers are respectively treated as a shared input and a shared output. A detailed statistic description of the dataset is provided in Podinovski (2022).

To evaluate the performance of all universities, we first applied of the multicomponent additive model (13). As a result, a total number of 62 universities were identified as Pareto-efficient, with an optimal objective value of zero. For the remaining 40 inefficient universities, we calculated their efficient targets using the additive-based formulation (14). It should be noted that, according to the additive model's objective function, these efficient targets represent the maximum L1 distance from the corresponding inefficient units. The optimal values of model (13) for inefficient universities, together with the corresponding input and output slacks are summarized in Table 1.

In the next stage, we applied the closest target model (22) to determine Pareto-efficient targets with minimum distance to each inefficient university in the multicomponent input-output space. Similarly, the optimal values of model (22) for inefficient universities, together with the associated slacks for all inputs and outputs are represented in Table 2. As expected, for all universities that were already Pareto-efficient, the optimal value of both models (13) and (22) are zero, indicating no required adjustments. Therefore, Pareto-efficient units have not been appeared neither in Table 1, nor in Table 2.

Table 1. Optimal Slacks in Additive model (13)

	Opt. Val.	ExpMed	UGMed	PGMed	ExpSci	UGSi	PGSci	ExpSoc	UGSoc	PGSoc	ExpCent	Papers
U2	30631.13			59.86	10813.75		340.96		350.35		19066.21	
U5	16806.03				5204.15	1258.83		4670.67			5672.38	
U9	20437.41				2626.09		115.82	7877.44		123.55	9651.82	42.70
U11	30675.45	506.82			15748.82			10716.98	792.15		2910.67	
U14	32586.94	2436.02		39.42	10556.44	199.26		11992.58			7363.21	
U16	6731.33		3.02	27.74	3605.91	1955.60	142.29				996.77	
U19	33046.18				6724.91	358.76		13332.20			12630.31	
U22	12702.84				1855.25		194.04	5264.28			5389.28	
U24	116341.86				17072.71	1940.10	3.56	24401.63			72923.87	
U26	26678.08					2750.32		7697.90		620.22	15609.65	
U27	30413.31				7884.47	96.50		12584.93			9847.41	
U35	20426.74				2141.14	731.10		7977.61			9576.89	
U36	17631.85						219.41	2780.52			14631.91	
U37	21261.01					313.86				404.81	20542.34	
U42	16645.00						230.77	5146.90	578.57		10688.76	
U45	44058.17					991.18		6620.61		1186.25	35260.13	
U47	51993.23				11129.63			11775.14	1930.23		27158.23	
U49	37545.71				2171.15	2630.42		3123.59			29620.56	
U50	138229.13			139.94		1120.92		18010.41		8.25	118949.62	
U51	2279.49						37.94	811.84	95.32		1334.38	
U53	31889.03		12.36	11.82	2348.70	1214.72	263.89	9231.96		100.22	18705.36	
U58	21709.88				2536.10	791.78		5640.05			12741.96	
U61	30167.22			34.45		568.20	32.95		24.57	73.41	29433.65	
U64	35463.58				4733.10	1086.26		4934.34			24709.88	
U67	57800.72				13802.79	845.85		24420.79			18731.30	
U72	53397.41				9432.49	1049.31	276.56	13511.15			29127.90	
U73	950431.30		830.66	1457.34	28766.05	3503.34	2592.58	46211.18	1735.65	4492.85	860841.67	
U77	56632.22					2174.32	190.88	6817.93	1542.16		45906.93	
U78	32802.30					2257.58		18309.41		639.26	11596.06	
U79	35345.01					1008.60	297.22	2931.86		1098.91	30008.43	
U81	59318.22							8807.19			50511.03	
U82	38292.23				3061.01			18052.29	785.92	113.36	16279.66	
U84	50358.76				9870.95		130.03	9086.93		432.75	30781.21	56.89
U87	12464.08				2086.08	496.53		7282.13		404.55	2189.19	5.60
U88	39734.13	600.00			8064.70	1808.61	56.39	10308.34			18896.09	
U90	40538.91			42.44	8092.63	1545.81	1002.63	19279.81			10575.59	
U97	5100.81						30.14	1981.74		25.16	3063.77	
U98	29333.65		6.64		11036.45		778.31	4789.10	248.27		12474.88	
U99	25020.40				6780.48			7476.42	713.35		10050.16	
U101	9565.18					134.99		279.45			8594.91	

Table 2. Optimal Slacks in Closest Target Model (22)

	Opt. Val.	ExpMed	UGMed	PGMed	ExpSci	UGSi	PGSci	ExpSoc	UGSoc	PGSoc	ExpCent	Papers
U2	2489.11		65.13	40.97		1456.30	926.71					
U5	415.59		14.13			392.21	9.25					
U9	1688.79					343.83	370.35		607.80	256.90		109.91
U11	10755.98	535.06	29.11		8312.76	433.36			1095.39	247.36		102.94
U14	1880.04	665.17				385.36	336.32			79.43		413.77
U16	4829.63		3.02	28.66	3083.91	1595.90	110.11			4.91		3.11
U19	1217.52		2.67				427.49		549.28			238.08
U22	681.24		5.50	0.18			205.50		405.34			64.73
U24	21396.17		2.93	12.06	5981.51	1572.57	902.86	12489.71		434.54		
U26	913.57		0.13	4.98			59.04			849.42		
U27	1026.28			1.76		72.26				478.19		474.07
U35	115.42		7.74	1.98			65.30					40.39
U36	37.53		0.38	4.83			32.32					
U37	87.01			2.11			6.08			6.50		72.33
U42	1642.51		33.88				691.37		549.70	367.56		
U45	1565.50		5.31	14.06			15.57			1530.56		
U47	2130.14		16.81				323.02		1082.59	211.98		495.74
U49	2130.14		16.81				323.02		1082.59	211.98		495.74
U50	14729.58			212.55		1639.26	393.14	9332.08		1578.30	892.47	681.79
U51	243.84			0.31			53.21		190.32			
U53	1047.77		20.75	10.38		66.51	389.59			313.07		247.48
U58	87.49		14.20	1.32								71.96
U61	662.32			33.99		407.03	85.39			24.80		111.11
U64	781.56			4.05			137.16			495.19		145.16
U67	4241.98		6.27		1248.22	1104.73	252.81		926.72	660.07		43.17
U72	1602.58		1.44	7.27		423.50	317.00			805.33		48.04

U73	743727.43	2553.34	877.26	1160.84	21018.94	3057.72	2216.60	43770.72	1160.15	3674.53	664237.34	
U77	577.48			26.38		543.53	0.16			7.41		
U78	887.89		1.09	4.19		56.05	19.03			742.15		65.40
U79	1245.15			14.93		796.29				433.93		
U81	490.76		24.36	5.66			135.22		177.04			148.48
U82	4102.53			0.06					2280.52	1537.27		284.69
U84	2564.89			7.56		1070.29	415.28			633.57		438.20
U87	3515.25				784.75		46.27	2188.10		415.84		80.29
U88	1604.00	73.67	31.20	5.67		771.29	91.04			86.25		544.88
U90	2740.75			68.60		828.71	942.80			900.64		
U97	91.98		1.76				76.81					13.41
U98	2563.83		6.86		712.78	631.89	877.64					334.67
U99	994.50			3.35			164.00		447.13	247.34		132.69
U101	260.73			2.88						257.84		

Conclusion and Discussion

Target setting is generally considered a critical issue for managers and decision-makers, as it translates efficiency assessment into actionable managerial guidance. By identifying feasible and attainable performance benchmarks, target setting bridges the gap between diagnostic efficiency scores and practical improvement strategies. In particular, the determination of closest targets is more critical for inefficient units, since such targets require minimal adjustments in inputs and/or outputs while remaining consistent with the underlying production technology. Closest targets enhance realism and implementability, reduce resistance to change, and provide clearer operational directions, especially in complex multicomponent settings where large simultaneous adjustments may be impractical or costly. As investigated in this paper, identification of closest targets corresponds to solving a minimal-distance projection problem that preserves feasibility and consistency with the multi-component technology set. By focusing on closest targets, managers can prioritize incremental improvements, allocate resources more effectively, and design incentive schemes, which is suggested as possible directions for future researches. Moreover, to develop relevant approaches for ranking efficient DMUs, and also to investigate the issue of inverse DEA models in the context of multicomponent production technologies can be considered in future studies.

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تعیین هدف در فناوری‌های تولید با فرآیندهای چند جزئی

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اطلاعات مقاله

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چکیده

در این مقاله، ما فناوری‌های تولید غیرپارامتری با فرآیندهای تولید چند جزئی را در نظر می‌گیریم، که در آن هر جزء از ورودی‌های خاص و مشترک برای تولید خروجی‌های خاص و مشترک استفاده می‌کند. با بررسی مسئله تعیین هدف در این فناوری‌ها، نشان می‌دهیم که چگونه می‌توان با استفاده از مدل‌های تحلیل پوششی پایه و تحلیل پوششی داده‌های ضریب‌دار، معیارهای کارآمدی را برای واحدهای ناکارآمد در فناوری‌های تولید چند جزئی به دست آورد. علاوه بر این، ما مرز کارآمدی فناوری چند جزئی را فرموله می‌کنیم و یک مدل بهینه‌سازی عدد صحیح مختلط را توسعه می‌دهیم که نزدیک‌ترین اهداف را برای هر فعالیت ناکارآمد به دست می‌آورد. مدل‌های پیشنهادی در نهایت بر روی یک مجموعه داده واقعی شامل ۱۰۲ دانشگاه دولتی در بریتانیا نشان داده می‌شوند.

کلیدواژگان: *پارتوکارا، کوژبودگی، نقطه فرافکنی، نزدیک‌ترین هدف.*

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